

Renormalization of the S parameter in holographic models of electroweak symmetry breaking

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ABSTRACT: We show that the S parameter is not finite in theories of electroweak symmetry breaking in a slice of anti-de Sitter five-dimensional space, with the light fermions localized in the ultraviolet. We compute the one-loop contributions to S from the Higgs sector and show that they are logarithmically dependent on the cutoff of the theory. We discuss the renormalization of S , as well as the implications for bounds from electroweak precision measurements on these models. We argue that, although in principle the choice of renormalization condition could eliminate the S parameter constraint, a more consistent condition would still result in a large and positive S . On the other hand, we show that the dependence on the Higgs mass in S can be entirely eliminated by the renormalization procedure, making it impossible in these theories to extract a Higgs mass bound from electroweak precision constraints.

KEYWORDS: Beyond Standard Model, Technicolor and Composite Models.

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1. Introduction

Although the standard model (SM) is an extremely successful description of the electroweak interactions, the instability of the weak scale under radiative corrections leads us to believe that there should be physics beyond the SM at an energy scale not far beyond the TeV. The origin of electroweak symmetry breaking (EWSB) as well as of fermion masses, might be associated with this new dynamics. A proposal for stabilizing the weak scale using a theory with one compact extra dimension with a non-factorizable, Anti de Sitter metric [1], the Randall-Sundrum (RS) model, can be thought of as dual to a strongly coupled four-dimensional theory with a large number of colors [2]. The slice of AdS₅ is defined by an ultra-violet (UV) fixed point located at the Planck scale, M_P , and an infra-red (IR) one, with an exponentially suppressed scale which is identified as the TeV scale. The 5D metric in conformal coordinates is given by:

$$ds^2 = \left(\frac{1}{kz}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \tag{1.1}$$

where k is the AdS₅ curvature. This spacetime has two 4D boundaries at $z_0 = 1/k \sim 1/M_P$ and $z_1 \sim 1/\text{TeV}$, respectively the UV and IR boundaries.

In order to stabilize the weak scale, the Higgs field must be localized at or near the TeV brane. This is not the case with the rest of the fields, which can then propagate in the AdS₅ bulk. The theories built this way, bulk AdS₅ models, not only avoid potentially troublesome higher dimensional operators suppressed only by the TeV scale, but also allow for a natural explanation of the fermion mass hierarchy [3–5].

There are several possibilities for building models of electroweak symmetry breaking in AdS₅. The basic elements for building a successful theory include the choice of the bulk gauge group, the zero-mode fermion localization and the dynamical mechanism for localizing the Higgs field on or near the TeV brane. The bulk gauge symmetry must be enlarged with respect to the SM in order to include isospin symmetry and avoid tree level contributions to the T parameter. A minimal extension [6] is $SU(2)_L \times SU(2)_R \times U(1)_X$, broken by boundary conditions either to the SM gauge group $SU(2)_L \times U(1)_Y$ or directly to $U(1)_{EM}$ as in Higgsless models [7]. In order to naturally address the fermion mass hierarchy, light fermions must be localized close to the UV boundary, and heavier fermions, such as the top quark, must be localized towards the IR brane to have a significant Yukawa coupling to the Higgs field [3, 4]. Finally, the IR localization of the Higgs can be dynamically achieved in specific models of EWSB. For instance, in a Gauge-Higgs unification model [8, 9] in AdS₅, the Higgs arises from the A_5 components of a gauge field and is naturally localized towards the TeV brane, as required to solve the hierarchy problem; whereas the inclusion of a fourth-generation highly localized towards the IR brane can result in a condensation of some of the fourth-generation zero modes and therefore in a Higgs localized near the IR [10]. IR localization of the Higgs can even be achieved in soft-wall models without an IR brane, as it is shown in ref. [11] in a Gauge-Higgs unification model.

These bulk AdS₅ models of EWSB and fermion masses can be thought of as duals of some strongly coupled 4D theory. They all share a common problem regarding electroweak precision constraints: a tree-level S parameter. This is approximately given by [6, 12]

$$S_{\text{tree}} \simeq 2\pi v^2 z_1^2, \tag{1.2}$$

where $v \simeq 246$ GeV is the vacuum expectation value of the Higgs field and we took the limit $v z_1 \ll 1$.¹ For a TeV scale IR brane this results in $S_{\text{tree}} \simeq 0.3$, in contradiction with current electroweak constraints [13]. It is possible to avoid this problem by de-localizing fermions [6, 14]. But in doing so, we would lose one of the most interesting features of these theories, namely a natural way of generating the fermion mass hierarchy. In this paper, we will restrict ourselves to models with light fermions localized near the UV boundary.

The presence of the tree-level S parameter in all bulk AdS₅ models poses a very stringent constraint on them. It suggests that it would be of interest to study the loop contributions to it. In this paper we compute the one loop contributions to the S parameter in these models coming from loops involving the Higgs sector. We will show that the one loop contributions to S are logarithmically divergent and therefore require that S be properly renormalized. We argue that similar divergences are expected in the fermion and gauge boson loops. The fact that S is logarithmically sensitive to the cutoff should not be completely surprising. From the point of view of the 5D theory, this is the cutoff of the non-renormalizable theory, properly warped down. On the other hand, in the 4D holographic picture, this cutoff corresponds to the matching of the low energy effective theory and the 4D strongly coupled CFT. A more subtle question is the choice of a renormalization

¹In Gauge-Higgs unification models there is typically an additional suppression given by $(v/f_\pi)^2$, the ratio of the Higgs VEV to the symmetry breaking scale [9].

condition for S . Although the logarithmic divergence is sub-dominant in the large N expansion, it could be numerically sizable. Furthermore, the renormalization procedure introduces a scale dependence in the S parameter. All in all, the use of the S parameter as a tight constraint on the mass scale of the Kaluza-Klein (KK) excitations as well as on the Higgs mass must be reassessed.

The plan for the rest of the paper is as follows: in the next section we present the setup of AdS₅ bulk models and derive the low energy effective theory obtained after integrating out the 5D bulk; in section 3 we compute the one-loop contributions of the Higgs sector to the S parameter in the effective theory and discuss the renormalization procedure. Finally, in section 4 we discuss our results and conclude.

2. Electroweak symmetry breaking in AdS₅

We consider a 5D model in a slice of AdS, with the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_X$, broken to the SM in the UV boundary. The 5D action is given by:

$$S = \int d^4x \int dz \sqrt{g} \left[-\frac{1}{4} L_{MN}^a L^{aMN} - \frac{1}{4} R_{MN}^a R^{aMN} - \frac{1}{4} X_{MN} X^{MN} \right], \quad (2.1)$$

where L_{MN}^a , R_{MN}^a and X_{MN} are the $SU(2)_L$, $SU(2)_R$ and $U(1)_X$ field strengths, and g is the determinant of the metric.

The Higgs field transforms as $(\mathbf{2}, \mathbf{2})_0$ under the gauge symmetry,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + i\phi_3 & i(\phi_1 - i\phi_2) \\ i(\phi_1 + i\phi_2) & v + h - i\phi_3 \end{pmatrix} \quad (2.2)$$

and it is localized near the IR brane by some suitable mechanism, such as in Gauge-Higgs unification [9] or the condensation of a fourth-generation zero-mode fermion [10]. Here it suffices to assume an effective localization on the IR boundary as given by

$$S_{\text{IR}} = \int d^4x \int dz \delta(z - z_1) \sqrt{g_{\text{IR}}} \left[\frac{1}{2} \text{Tr} |D_\mu H|^2 - V(H) \right], \quad (2.3)$$

with g_{IR} the induced metric in the IR boundary and $V(H)$ the usual renormalizable Higgs potential. The covariant derivative acting on the scalar field is defined as:

$$D_\mu H = \partial_\mu H - ig_5 L_\mu H + i\tilde{g}_5 H R_\mu, \quad (2.4)$$

where g_5 and \tilde{g}_5 are the $SU(2)_L$ and $SU(2)_R$ 5D gauge couplings, respectively. As usual, in order to obtain a canonically normalized Higgs kinetic term, we rescale the Higgs field by $H \rightarrow (1/kz_1) H$.

2.1 The low energy effective theory

The presence of the 5D bulk affects the couplings of gauge bosons to the Higgs sector, as well as to fermions. In order to compute the one loop contributions to electroweak precision constraints, we will integrate out the 5D bulk and obtain a low energy theory containing

the zero-mode gauge bosons and the Higgs. We will use the holographic approach to obtain the resulting low energy effective theory, separating the UV degrees of freedom. This is useful since the UV boundary and the bulk respect different symmetries.

The Higgs VEV $\langle H \rangle = v\mathbf{I}/\sqrt{2}$ breaks the $SU(2)_L \times SU(2)_R$ symmetry down to $SU(2)_V$. Therefore it is convenient to work in the vector and axial-vector basis in the bulk, defined by

$$\begin{aligned} V_M &= \frac{1}{\sqrt{g_5^2 + \tilde{g}_5^2}} (\tilde{g}_5 L_M + g_5 R_M), \\ A_M &= \frac{1}{\sqrt{g_5^2 + \tilde{g}_5^2}} (g_5 L_M - \tilde{g}_5 R_M). \end{aligned} \quad (2.5)$$

We add the gauge fixing term:

$$\mathcal{L}_{GF}^V = -\frac{1}{kz \xi_V} \text{Tr} [\partial_\mu V_\mu - z \xi_V \partial_5 (V_5/z)]^2, \quad (2.6)$$

where ∂_5 is the derivative with respect to the z coordinate, and there will be similar terms for A_M and X_M . We will take the limit $\xi_{V,A,X} \rightarrow \infty$, and obtain $\partial_5(V_5/(kz)) = \partial_5(A_5/(kz)) = \partial_5(X_5/(kz)) = 0$. After integration by parts the quadratic term for V_μ in the 5D Lagrangian is

$$\mathcal{L} = \frac{1}{kz} \text{Tr} \{ V_\mu [(\partial^2 - z \partial_5(1/z) \partial_5) \eta_{\mu\nu} - \partial_\mu \partial_\nu] V_\nu \} + \dots, \quad (2.7)$$

and similarly for A_μ and X_μ .

Also left from the integration by parts are the following boundary terms:

$$\mathcal{L}_{\text{bound}} = \frac{1}{kz} \text{Tr} [V_\mu \partial_5 V_\mu - 2V_\mu \partial_\mu V_5 + A_\mu \partial_5 A_\mu - 2A_\mu \partial_\mu A_5 + X_\mu \partial_5 X_\mu - 2X_\mu \partial_\mu X_5] \Big|_{z_0}^{z_1}. \quad (2.8)$$

Since the IR-localized Higgs acquires a VEV, its kinetic term mixes A_μ with the Nambu-Goldstone bosons (NGBs) ϕ_i ($i=1,2,3$). We then add an additional gauge fixing term on the IR boundary

$$\mathcal{L}_{GF,IR}^A = -\frac{1}{\xi_{A,IR}} \text{Tr} \left(\partial_\mu A_\mu - \frac{\xi_{A,IR}}{2} \sqrt{(g_5^2 + \tilde{g}_5^2)k} v \sigma^i \phi_i \right)^2 \Big|_{z_1}. \quad (2.9)$$

We choose $\xi_{A,IR} = 0$. We then solve the bulk equations of motion obtained from (2.7), with the following boundary conditions on the IR:

$$\partial_5 V_\mu|_{z_1} = V_5|_{z_1} = \partial_5 X_\mu|_{z_1} = X_5|_{z_1} = 0, \quad (2.10)$$

$$\left(\frac{1}{kz} \partial_5 + \frac{g_5^2 + \tilde{g}_5^2}{4} v^2 \right) A_\mu|_{z_1} = A_5|_{z_1} = 0. \quad (2.11)$$

The solutions can be written as

$$\begin{aligned} V_\mu(p, z) &= \sqrt{k} V_\mu^0(p) f_V(p, z), & A_\mu(p, z) &= \sqrt{k} A_\mu^0(p) f_A(p, z) \\ X_\mu(p, z) &= \sqrt{k} X_\mu^0(p) f_V(p, z), \end{aligned} \quad (2.12)$$

with $f_{V,A}$ defined by:

$$f_V(p, z) = \frac{z(J_1(pz)Y_0(pz_1) - Y_1(pz)J_0(pz_1))}{z_0(J_1(pz_0)Y_0(pz_1) - Y_1(pz_0)J_0(pz_1))}, \quad (2.13)$$

$$f_A(p, z) = \frac{z[J_1(pz)(pY_0(pz_1) + m_1Y_1(pz_1)) - Y_1(pz)(pJ_0(pz_1) + m_1J_1(pz_1))]}{z_0[J_1(pz_0)(pY_0(pz_1) + m_1Y_1(pz_1)) - Y_1(pz_0)(pJ_0(pz_1) + m_1J_1(pz_1))]}, \quad (2.14)$$

where

$$m_1 = (g_5^2 + \tilde{g}_5^2)kv^2z_1/4, \quad (2.15)$$

and where $V_\mu^0(p) = V_\mu(p, z_0)/\sqrt{k}$ is the UV-boundary value of the V_μ field, with analogous definitions for the UV fields of A_μ and X_μ . In what follows we will drop the index 0, and will refer to the UV fields simply as V_μ , A_μ and X_μ .

The low energy effective theory can then be written in terms of the UV fields and the IR-localized Higgs. It is obtained by substituting the solutions for the UV fields back into the action. The resulting low energy effective theory comes from the UV boundary terms, and describes the interactions of the “elementary” fields coupled to the IR-localized Higgs. These interactions encode the effects of the bulk that was integrated out. In order to have the SM gauge field content at low energies, we choose the dynamical fields at low energy to be the $SU(2)_L \times U(1)_Y$ gauge fields

$$L_\mu^a, a = 1, 2, 3; \quad B_\mu = \frac{g_{5X}R_\mu^3 + \tilde{g}_5X_\mu}{\sqrt{\tilde{g}_5^2 + g_{5X}^2}}, \quad (2.16)$$

with g_{5X} the 5D $U(1)_X$ gauge coupling, whereas the other gauge fields in the UV

$$R_\mu^a, a = 1, 2; \quad S_\mu = \frac{\tilde{g}_5R_\mu^3 - g_{5X}X_\mu}{\sqrt{\tilde{g}_5^2 + g_{5X}^2}}, \quad (2.17)$$

are given Dirilchet boundary conditions and are not present in the effective theory. We define the 5D hypercharge coupling constant by:

$$g_{5Y} = \frac{\tilde{g}_5g_{5X}}{\sqrt{\tilde{g}_5^2 + g_{5X}^2}}. \quad (2.18)$$

After integrating out the bulk gauge fields, the momentum-space quadratic terms in the effective Lagrangian are

$$\begin{aligned} \mathcal{L}_{\text{eff}}^2 = & \frac{P_{\mu\nu}}{2} [L_\mu^a \Pi_L(p^2)L_\nu^a + 2L_\mu^3 \Pi_{3B}(p^2)B_\nu + B_\mu \Pi_B(p^2)B_\nu] \\ & - \frac{1}{2}h(p^2 + m_h^2)h - \frac{1}{2}\phi_i p^2 \phi_i, \end{aligned} \quad (2.19)$$

where the correlators Π_L, Π_{3B} and Π_B are given by

$$\Pi_L(p^2) = \frac{\tilde{g}_5^2 \Pi_V + g_5^2 \Pi_A}{g_5^2 + \tilde{g}_5^2}, \quad (2.20)$$

$$\Pi_{3B}(p^2) = \frac{g_5 \tilde{g}_5 g_{5X}}{(g_5^2 + \tilde{g}_5^2) \sqrt{\tilde{g}_5^2 + g_{5X}^2}} (\Pi_V - \Pi_A), \quad (2.21)$$

$$\Pi_B(p^2) = \frac{(g_5^2 g_{5X}^2 + g_5^2 \tilde{g}_5^2 + \tilde{g}_5^4) \Pi_V + \tilde{g}_5^2 g_{5X}^2 \Pi_A}{(g_5^2 + \tilde{g}_5^2)(\tilde{g}_5^2 + g_{5X}^2)}, \quad (2.22)$$

$$(2.23)$$

and $\Pi_{V,A}(p^2)$ are the vector and axial correlators, defined as

$$\Pi_V(p^2) = -\frac{p[J_0(pz_0)Y_0(pz_1) - Y_0(pz_0)J_0(pz_1)]}{z_0[J_1(pz_0)Y_0(pz_1) - J_0(pz_1)Y_1(pz_0)]}, \quad (2.24)$$

$$\Pi_A(p^2) = -\frac{pJ_0(pz_0)[pY_0(pz_1) + m_1Y_1(pz_1)] - pY_0(pz_0)[pJ_0(pz_1) + m_1J_1(pz_1)]}{z_0J_1(pz_0)[pY_0(pz_1) + m_1Y_1(pz_1)] - z_0Y_1(pz_0)[pJ_0(pz_1) + m_1J_1(pz_1)]}. \quad (2.25)$$

The tree-level contribution to $S = -16\pi/(gg')\Pi'_{3B}(0)$, can already be obtained from the momentum-dependent correlator in (2.21). Defining the 4D gauge couplings by

$$g_5^2 \simeq \frac{1}{k} \log \frac{z_1}{z_0} g^2, \quad g_{5Y}^2 \simeq \frac{1}{k} \log \frac{z_1}{z_0} g'^2, \quad (2.26)$$

where we have discarded terms of order $\mathcal{O}(vz_1)^2$, one obtains

$$S_{\text{tree}} = 4\pi v^2 z_1^2 \frac{32 + 3(g_5^2 + g_{5Y}^2)v^2 z_1^2}{(8 + (g_5^2 + g_{5Y}^2)v^2 z_1^2)^2} \simeq 2\pi v^2 z_1^2, \quad (2.27)$$

where we have taken the limit $vz_1 \ll 1$ to obtain the last expression.

In the absence of new terms localized on the UV boundary, the propagators of the UV fields are given by the inverse of the correlators. In the diagonal basis $\{\gamma_\mu, Z_\mu\}$ we have

$$\gamma_\mu = \frac{\tilde{g}_5 g_{5X} L_\mu^3 + g_5 \sqrt{\tilde{g}_5^2 + g_{5X}^2} B_\mu}{[g_5^2(\tilde{g}_5^2 + g_{5X}^2) + \tilde{g}_5^2 g_{5X}^2]^{1/2}}, \quad Z_\mu = \frac{g_5 \sqrt{\tilde{g}_5^2 + g_{5X}^2} L_\mu^3 - \tilde{g}_5 g_{5X} B_\mu}{[g_5^2(\tilde{g}_5^2 + g_{5X}^2) + \tilde{g}_5^2 g_{5X}^2]^{1/2}}, \quad (2.28)$$

with correlators given by

$$\Pi_\gamma = \Pi_V, \quad \Pi_Z = \frac{\tilde{g}_5^4 \Pi_V + (\tilde{g}_5^2 g_{5X}^2 + g_5^2 \tilde{g}_5^2 + g_5^2 g_{5X}^2) \Pi_A}{(g_5^2 + \tilde{g}_5^2)(\tilde{g}_5^2 + g_{5X}^2)}. \quad (2.29)$$

Finally, the spectrum of vector resonances, corresponding to the KK spectrum, is given by the zeroes of $\Pi_\gamma(p^2)$, $\Pi_Z(p^2)$ and $\Pi_L(p^2)$.

2.2 Gauge-Higgs interactions

In order to compute the one-loop corrections to the S parameter coming from the Higgs sector, we need the interactions of the gauge bosons and the Higgs in the low energy effective theory. The interactions of interest are the cubic interactions described by

$$\begin{aligned} \mathcal{L}_{\text{eff}}^3 &= \frac{g}{2} L^{1\mu}(p) [c_A(p) h \overleftrightarrow{\partial}_\mu \phi_1 + c_V(p) \phi_3 \overleftrightarrow{\partial}_\mu \phi_2] \\ &+ \frac{g}{2} L^{3\mu}(p) [c_A(p) h \overleftrightarrow{\partial}_\mu \phi_3 + c_V(p) \phi_2 \overleftrightarrow{\partial}_\mu \phi_1] \\ &+ \frac{g'}{2} B^\mu(p) [-c_A(p) h \overleftrightarrow{\partial}_\mu \phi_3 + \tilde{c}_V(p) \phi_2 \overleftrightarrow{\partial}_\mu \phi_1], \end{aligned} \quad (2.30)$$

where $c_A(p)$, $c_V(p)$ and $\tilde{c}_V(p)$ have a non-trivial dependence with momentum and are defined by

$$c_V(p) = \frac{2\tilde{g}_5^2 f_V(p, z) + (g_5^2 - \tilde{g}_5^2) f_A(p, z)}{(g_5^2 + \tilde{g}_5^2)} \Big|_{z_1}, \quad \tilde{c}_V(p) = \frac{2g_5^2 f_V(p, z) + (\tilde{g}_5^2 - g_5^2) f_A(p, z)}{(g_5^2 + \tilde{g}_5^2)} \Big|_{z_1},$$

$$c_A(p) = f_A(p, z) \Big|_{z_1}. \quad (2.31)$$

Taking the limit of $z_1 \rightarrow z_0$ we recover the SM couplings, with $c_V = \tilde{c}_V = c_A = 1$, but for finite $(z_1 - z_0) \sim 1/\text{TeV}$ the gauge-Higgs couplings are modified with respect to their SM values. In particular, the fact that $c_V(p)\tilde{c}_V(p) \neq c_A^2(p)$ in (2.31) will result in divergences in the one loop calculation of the S parameter.

We will also need the quartic interactions given by

$$\begin{aligned} \mathcal{L}_{\text{eff}}^4 = & \frac{g^2}{8} L_\mu^1(p) L_1^\mu(k) \left\{ [(2v+h)h + \phi_1^2] c_A(p) c_A(k) + (\phi_2^2 + \phi_3^2) c_V(p) c_V(k) \right\} \\ & + \frac{g^2}{8} L_\mu^3(p) L_3^\mu(k) \left\{ [(2v+h)h + \phi_3^2] c_A(p) c_A(k) + (\phi_1^2 + \phi_2^2) c_V(p) c_V(k) \right\} \\ & + \frac{g^2}{8} B_\mu(p) B^\mu(k) \left\{ [(2v+h)h + \phi_3^2] c_A(p) c_A(k) + (\phi_1^2 + \phi_2^2) \tilde{c}_V(p) \tilde{c}_V(k) \right\} \\ & + \frac{gg'}{4} B_\mu(p) L_1^\mu(k) (v+h) \phi_2 \left(\frac{g_5^2 c_A(k)}{g_5^2 + \tilde{g}_5^2} [c_V(p) + \tilde{c}_V(p)] + \frac{\tilde{g}_5^2 c_A(p)}{g_5^2 + \tilde{g}_5^2} [c_V(k) + \tilde{c}_V(k)] \right) \\ & + \frac{gg'}{4} B_\mu(p) L_3^\mu(k) \left\{ - [(2v+h)h + \phi_3^2] c_A(p) c_A(k) + (\phi_1^2 + \phi_2^2) \tilde{c}_V(p) c_V(k) \right\}, \quad (2.32) \end{aligned}$$

A few comments are in order. First, the fact that the gauge-Higgs couplings are modified due to the presence of the KK resonances is not particular of the specific symmetry considered. For instance, had we consider instead $SU(2)_L \times U(1)_Y$ we would have also obtained shifts in the couplings which are not the same for the different components of the Higgs, and in particular we would still have $c_V(p)\tilde{c}_V(p) \neq c_A^2(p)$. Secondly, if we allow for a Higgs bulk profile, $f_H(z)$, the couplings c_V , \tilde{c}_V and c_A would depend on this profile. However, since the Higgs must be quite localized near the IR brane, the approximation made here (perfect IR localization) should capture the essence of the effects up to small corrections. Also, the couplings in (2.31) entering in the cubic and quartic interactions of (2.30) and (2.32) introduce an additional dependence on the external momentum.

Finally, the effective low energy theory is obtained by integrating the 5D bulk, i.e. it is taking into account the effects of all the KK modes. It is also interesting to obtain the effective couplings c_V , \tilde{c}_V and c_A by integrating one or two KK modes and see how rapidly the process converges. This can be seen in table 1, where we show the effective couplings at zero momentum for the full 5D bulk integration, the case when only one KK mode is integrated out and taken into account, and finally the results obtained with the first two KK modes integrated out. These results are approximated (for instance, we assume $(M_{KK}^{(1)})^2 \simeq 6/z_1^2$, and $(M_{KK}^{(2)})^2 \simeq 30/z_1^2$), but already give a sense of the convergence of the procedure. The effect of the KK modes comes essentially from the mixing of the axial-vector combination with the zero-mode gauge bosons, triggered by the Higgs VEV. We conclude that the KK picture is not a bad approximation and the first KK modes do

Eff. Couplings	Holography	1st KK	1st+ 2nd KKs
$c_V(0)$	$\frac{2\tilde{g}_5^2 kv^2 z_1^2 + 8}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 8}$	$\frac{2\tilde{g}_5^2 kv^2 z_1^2 + 12}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 12}$	$\frac{2\tilde{g}_5^2 kv^2 z_1^2 + 10}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 10}$
$c_A(0)$	$\frac{8}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 8}$	$\frac{12}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 12}$	$\frac{10}{(\tilde{g}_5^2 + g_5^2)kv^2 z_1^2 + 10}$

Table 1: Effective couplings computed integrating out the 5D bulk, only the first KK resonance, and only the first and second KK resonances. The remaining coupling is given by $\tilde{c}_V^2 = 2 - c_V^2$.

capture the correct physics in approximate magnitude and sign. However, and since it is fairly straightforward to obtain the full 5D bulk integration of the holographic picture, we will use the full result obtained in sections 2.1 and 2.2.

3. Higgs contributions to electroweak parameters

Since the couplings between the Higgs sector and the SM gauge bosons are modified by the presence of the 5D bulk, we expect effects in the electroweak parameters with respect to the SM. In the SM, the Higgs contributions to the S and T parameters are finite because the potentially divergent terms cancel when we add the different diagrams. As we will show, in the present model the Higgs contribution to S is cutoff sensitive. In the effective theory described in the previous section, the shifts in the Higgs couplings to the SM gauge bosons will result in additional contributions to T and S and in particular, in divergent contributions to S . Due to the custodial symmetry, there is no tree-level contribution to T , as can be seen from eq. (2.19), since $\Pi_{11} = \Pi_{33} = \Pi_L$ at this order. In the appendix we explicitly show that the one-loop contribution to T is finite, as expected also from the custodial symmetry, as well as from the absence of a counter-term. In what follows we present the calculation of the one loop Higgs contributions to S in the low energy effective theory.

3.1 Contribution to S in the effective theory

The one loop contributions of the Higgs sector to the S parameter are those depicted in the Feynman diagrams of figure 1. The dots denote the effective couplings in eqs. (2.30) and (2.32), obtained by integrating out the 5D bulk.

Although the contributions to one-loop self-energies coming from the gauge sector are generically gauge dependent, the contributions from the diagrams in figure 1 to *oblique electroweak corrections* are separately gauge-invariant. In general, the gauge dependence of gauge-boson self-energies is cancelled by vertex and box diagrams which induce pinch propagator-like contributions [15]. However, the pinch contributions that affect the diagrams of figure 1 are *non-oblique* [16], implying that the oblique pieces of these diagrams are gauge invariant. Therefore, the contributions of the diagrams of figure 1 to oblique electroweak parameters are separately gauge invariant.

Not all contributions from the diagrams in figure 1 should be considered as contributions to S . Some of them are renormalizing the Higgs VEV. In order to see how to identify these pieces, it is instructive to first turn to the tree-level contribution to S , S_{tree} , as shown in (2.27). Since the Higgs is localized on the IR brane, the effects of EWSB must go

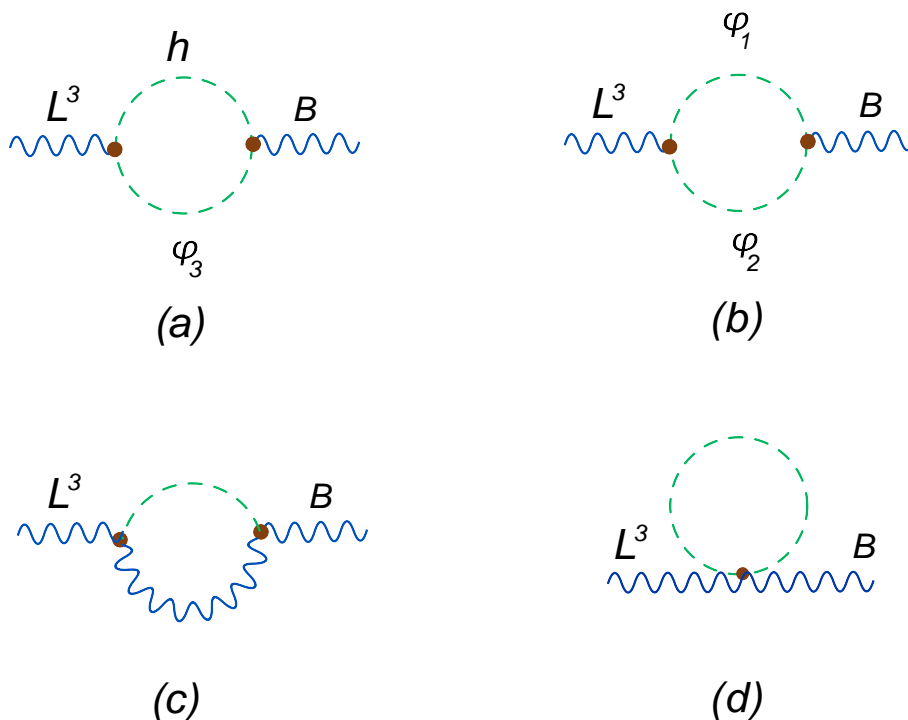


Figure 1: One-loop Feynman diagrams contributing to the S parameter involving the Higgs sector. The large dots stand for the effective couplings of eqs. (2.30) and (2.32).

through the 5D bulk in order to be felt by the UV fields. In particular, the mixing between B and L_3 caused by the Higgs VEV in the IR brane, picks up a momentum dependence in the bulk, resulting in *kinetic* mixing, and therefore in a contribution to S given by S_{tree} . The loop contributions in figure 1 are also IR-localized. External momentum dependence arises from either external momentum in the loop, or the momentum dependent coefficients $c_A(p)$, $c_V(p)$ and $\tilde{c}_V(p)$ appearing in the Higgs couplings to gauge bosons in the effective theory. The latter, is the momentum dependence that the IR-localized loops acquire when going from the IR brane to the UV, where the elementary gauge bosons are. These contributions do not have external momentum dependence themselves in the IR, and they correspond to various renormalizations, such as the renormalization of v appearing in S_{tree} in eq. (2.27). Thus, as a general rule, genuine contributions to the S parameter are those with external momentum actually flowing through the loop. This amounts to computing the loop diagrams with the effective couplings $c_A(p)$, $c_V(p)$ and $\tilde{c}_V(p)$ evaluated at zero external momentum. This means, for instance, that the diagram 1-(d) will not contribute to S , but that the momentum dependence from the couplings results in a renormalization of v appearing in S_{tree} .

The diagram 1-(c) gives a finite contribution to S . This can be seen by noticing that the corresponding loop diagram gives

$$i\Pi_{3B}^{1(c)}(p^2) = -\frac{(g^2 + g'^2)^2}{4} v^2 c_w s_w c_A^2(p) \int \frac{d^4 k}{(2\pi)^2} c_A^2(p-k) G_h(k) G_Z(p-k), \quad (3.1)$$

corresponding to the $g_{\mu\nu}$ coefficient of the diagrams with L^3 and B inside the loop, and where c_w and s_w stand for the cosine and sine of the Weinberg angle respectively. In the SM limit, $c_A \rightarrow 1$, these loop diagrams result in finite contributions to S since their derivatives with respect to the external momentum are finite, even if the vacuum polarizations themselves are divergent. In the present case, however, the factor $c_A(q-k)$ regulates the vacuum polarization itself, since in the large momentum limit $c_A(k) \sim e^{-kz_1}$. As a consequence, the contributions of figure 1-(c) to S are not only finite but further suppressed. We just denote them as $S^{1-(c)}$ for the remainder of the paper.

The one loop contributions that do result in divergences in the S parameter are those from diagrams 1-(a) and 1-(b). In the SM, these diagrams are responsible for the m_h dependence in S and are therefore the main source of bounds on m_h from electroweak precision bounds. Using dimensional regularization we obtain:

$$\begin{aligned}
 S_{\text{loop}}^H &= \frac{1}{12\pi} (N_\epsilon - 1) [c_V(0)\tilde{c}_V(0) - c_A^2(0)] \\
 &+ \frac{1}{2\pi} \int_0^1 dx(1-x)x \left[c_A^2(0) \ln\left(\frac{\Delta}{\mu^2}\right) - c_V(0)\tilde{c}_V(0) \ln\left(\frac{M_W^2}{\mu^2}\right) \right] \\
 &+ \text{finite terms},
 \end{aligned}
 \tag{3.2}$$

where

$$N_\epsilon \equiv \frac{2}{\epsilon} - \gamma + 1 + \ln 4\pi,
 \tag{3.3}$$

$$\Delta \equiv xm_h^2 + (1-x)M_Z^2,
 \tag{3.4}$$

$\epsilon = 4 - d$, μ is a renormalization scale, and the finite terms come from diagram 1-(c). The first term in (3.2) is divergent in the low energy effective theory that results from integrating out the 5D bulk. The second term² gives the m_h dependence to the S parameter. The SM limit corresponds to taking $(c_V, \tilde{c}_V, c_A) \rightarrow 1$. From (3.2) we see that in this limit the divergent term cancels, and the second term results in the Higgs contribution to S in the SM:

$$S_{\text{SM}}^H = (1/12\pi) \ln(m_h^2/M_W^2) + \dots.
 \tag{3.5}$$

Thus, the result of taking into account the effects of the 5D bulk (or of the strongly coupled sector) is twofold: it makes the S parameter UV-sensitive and it modifies its m_h dependence.

If we regularize the momentum integrals using a cutoff procedure, the divergence in (3.2) is logarithmic. In this case the contributions to S from the Higgs sector can be written as

$$S_{\text{loop}}^H = \frac{1}{12\pi} [c_V(0)\tilde{c}_V(0) - c_A^2(0)] \ln \frac{\Lambda^2}{\mu^2} + \text{finite terms},
 \tag{3.6}$$

where Λ is the cutoff of the low energy effective theory. This should be the local IR cutoff, which is warped down to the TeV scale $1/z_1$ from the Planck scale k . So we have

$$\Lambda \sim \frac{1}{z_1}.
 \tag{3.7}$$

²In this gauge the NGBs are massless at this order, and the M_Z and M_W dependence in (3.2) comes from other loops which are finite, such as the one in figure 1-(c).

We conclude that the S parameter in AdS₅ bulk theories is logarithmically divergent and therefore cutoff dependent. In order to remove this divergence, the S parameter must be renormalized by choosing a suitable renormalization condition.

3.2 Renormalization of the S parameter

The one loop calculation of the contributions to S from the Higgs sector fixes the divergent part of the counter-term in the renormalization procedure. However, it does not fix the finite parts, for which we need a renormalization condition. In order to illuminate the discussion we write the S parameter as

$$S = S_{\text{tree}} + \delta S + S_{\text{loop}}, \quad (3.8)$$

where δS is a counter-term. In general, it can be written as

$$\delta S = \delta S^{\text{div.}} + \delta S^{\text{finite}} \quad (3.9)$$

where $\delta S^{\text{div.}}$ cancels the divergence in (3.2), and the finite part δS^{finite} is only determined by the renormalization conditions. The resulting renormalized S parameter acquires a scale dependence and can be written as

$$S(\mu) = S(\mu_0) + \frac{1}{12\pi} [c_V(0)\tilde{c}_V(0) - c_A^2(0)] \ln\left(\frac{\mu^2}{\mu_0^2}\right), \quad (3.10)$$

where μ_0 is a reference scale and $S(\mu_0)$ is

$$S(\mu_0) = S_{\text{tree}} - \frac{1}{12\pi} [c_V(0)\tilde{c}_V(0) - c_A^2(0)] + S^{1(c)} + \delta S^{\text{finite}} + \frac{1}{2\pi} \int_0^1 dx x(1-x) \left[c_A^2(0) \ln\left(\frac{\Delta}{\mu_0^2}\right) - \tilde{c}_V(0)c_V(0) \ln\left(\frac{M_W^2}{\mu_0^2}\right) \right], \quad (3.11)$$

where Δ is defined in (3.4). As mentioned earlier, the last term in (3.11) contains the Higgs mass dependence of the S parameter.

In order to determine $S(\mu_0)$ we must choose a renormalization condition, which basically fixes δS^{finite} . In principle, this could be arbitrarily chosen, for instance to match the experimentally measured value of S at some energy scale, such as

$$S(\mu_0) = S^{\text{exp.}}(\mu_0 = M_Z). \quad (3.12)$$

However, given that $S^{\text{exp.}}(\mu_0 = M_Z) \lesssim 0.1$, the choice in (3.12) amounts to assume a rather efficient cancellation of S_{tree} against δS^{finite} as well as against the loop contributions. Although this choice does not result in a numerically fine-tuned cancellation, it would imply that the leading order contribution to S in the large N expansion,

$$S_{\text{tree}} \simeq \frac{O(1)}{\pi} N, \quad (3.13)$$

is not a good enough approximation and that the next order in N is equally important. In order to see this, we define the number of colors N in the 4D CFT in term of the 5D gauge couplings by

$$\frac{1}{N} = \frac{(g_5^2 + \tilde{g}_5^2)}{16\pi^2} k, \quad (3.14)$$

which reflects that the large N corresponds to the perturbative expansion in the 5D gauge theory. Thus, the loop diagrams considered here are suppressed contributions in the large N expansion. This would mean that a renormalization condition such as (3.12) implies that an $O(N)$ contribution such as S_{tree} is efficiently canceled by $O(1)$ contributions coming from loops, which may call into question our use of the large N expansion, i.e. our use of perturbation theory in the 5D theory.

As concrete examples, we can study two different limits. First we consider $m_1 z_1 \gg 1$, which corresponds to a heavy Higgs or nearly Higgsless scenario. Using (2.27), results in [7]

$$S_{\text{tree}} \sim \frac{3}{4} \frac{N}{\pi}. \quad (3.15)$$

To obtain the one-loop contribution in this limit we need the zero-momentum limit of c_V , \tilde{c}_V and c_A , which results in

$$\tilde{c}_V(0)c_V(0) = \frac{4 [(1 + m_1 z_1)\tilde{g}_5^2 + g_5^2] [(1 + m_1 z_1)g_5^2 + \tilde{g}_5^2]}{(2 + m_1 z_1)^2 (g_5^2 + \tilde{g}_5^2)^2} \quad (3.16)$$

$$c_A^2(0) = \frac{4}{(2 + m_1 z_1)^2}, \quad (3.17)$$

where m_1 is defined in (2.15). Then, taking $m_1 z_1 \gg 1$ and using (3.14) we can see that the loop contributions in (3.10) are of order $O(1)$ in the large N expansion. We obtain

$$S_{\text{loop}} \sim \frac{1}{12\pi} \ln \frac{\mu^2}{m_h^2}. \quad (3.18)$$

We can also consider the limit $m_1 z_1 \ll 1$, in which case we have

$$S_{\text{tree}} \sim 2\pi v^2 z_1^2, \quad (3.19)$$

$$S_{\text{loop}} \sim \frac{1}{N} \frac{\pi}{3} v^2 z_1^2 \ln \frac{\mu^2}{m_h^2}. \quad (3.20)$$

A more conservative renormalization condition would be

$$S(\mu_0) \sim S_{\text{tree}}, \quad (3.21)$$

which amounts to assume that there is no significant cancellation of the tree-level contribution from the counter-term or from loop corrections. With this choice, a large positive S parameter is still predicted, but the prediction cannot be made precise.

Although the renormalization condition in (3.21) avoids large cancellations of $O(N)$ and $O(1)$ contributions, the finite pieces of the counter-term could still affect significantly the loop contributions. In particular, the last term in (3.11) containing the information on the Higgs mass, can be affected by the renormalization condition since it is of $O(1)$, just as the counter-term is expected to be. We then conclude that in these theories the S parameter cannot be used to put a bound on the Higgs mass.

The ‘‘correct’’ renormalization condition might be somewhere in between these two extremes, i.e. there may be some cancellation of S_{tree} dictated by the unknown UV (or

CFT) physics. In any case, what is clear from our calculation is that the composite, strongly-coupled Higgs sector suffers shifts in its couplings to the SM gauge fields in such a way that the usual cancellations in the diagrams of figure 1-(a) and figure 1-(b) do not occur. Thus, this misalignment of the gauge-Higgs couplings with respect to their SM values, results in a dependence on the cutoff scale (in the 5D language), or the matching scale with the 4D CFT (in the 4D picture).

4. Discussion and conclusions

We have computed the one-loop contributions to the S parameter from the Higgs sector in bulk AdS₅ theories of EWSB. In these generic 5D setups we have used the minimal extension of the gauge group that protects isospin symmetry in the bulk, avoiding a tree-level T parameter. Our results show that the S parameter is UV-sensitive and therefore it must be renormalized. The appearance of divergences in S are a consequence of the misalignment between the gauge fields in the IR, where they interact with the IR-localized Higgs, and the UV fields which constitute the elementary degrees of freedom in terms of the holographic picture. This misalignment is produced by the 5D bulk between the IR and the UV branes, or the strong dynamics from the 4D CFT, and it occurs independently of the choice of bulk gauge symmetry. These divergences are then completely generic in bulk AdS₅ models of electroweak symmetry breaking. Their origin is fundamentally different from the logarithmic divergence found in ref. [17], which has origin in the mixing of the Higgs with a state resulting from the symmetry breaking pattern in that model. On the other hand, they are similar in spirit to the matching-scale dependence found in refs. [18] and [19] in a three-site Higgsless model.

It is also possible to understand the occurrence of these divergences in a generic operator analysis. For instance, the operator

$$\mathcal{O}_H = (H^\dagger H) |D_\mu H|^2 \tag{4.1}$$

contributes to S when inserted in one-loop diagrams. Its contribution is logarithmically divergent and results in

$$S_{\mathcal{O}_H} \sim -\frac{c_H v^2}{12\pi} \ln\left(\frac{\Lambda^2}{m_h^2}\right), \tag{4.2}$$

where c_H is the corresponding coefficient of \mathcal{O}_H . On the other hand, we can do the matching of this operator to the AdS₅ bulk theory. Expanding Π_L in eq. (2.19) to fourth order in v at zero momentum, we obtain that

$$c_H = -\frac{g_5^2 + \tilde{g}_5^2}{4} k z_1^2, \tag{4.3}$$

which results in a prediction consistent with (3.6). Thus, we see that the divergence in S is a generic feature in strongly coupled theories, rather than specific to AdS₅ bulk models.

Coming back to the AdS₅ bulk models discussed in the paper, the renormalized S parameter has a calculable scale dependence given in (3.10). We discussed the possible choices of renormalization conditions. Although in principle it is possible to choose an

arbitrary condition, so as to adjust the renormalized value of S to any desired value, we showed that asking for a significant cancellation of the tree-level value S_{tree} , which is of $O(N)$ in the large N expansion, might be unnatural if the expansion is to be trusted. However, and by the same argument, the Higgs mass dependence in S , which appears in (3.11), is of $O(1)$ and therefore can be naturally affected by a renormalization procedure triggered by $O(1)$ one-loop corrections. We then conclude that in these theories there is no bound on the Higgs mass that can be extracted from S .

We only computed the one-loop contributions from the Higgs sector. However, we also expect divergent contributions from fermions and gauge bosons. As we have shown, the divergences can be associated with the shifts in the couplings between the SM gauge fields localized in the UV and the composite fields localized towards the IR. Thus, we expect a similar effect for composite fermions. These are more model-dependent and we leave their study for future work.

We finally comment on the case where the light fermions are de-localized. The limit of exact de-localization, results in flat zero modes. In this limit [6, 14] there is no tree level S and we do not expect divergences in the loop contributions. In this case, since the SM fermions exactly correspond to the zero modes, the SM gauge fields are also de-localized and their couplings with the Higgs and fermions are canonical.

A. Higgs sector contribution to T

Here we compute the one-loop contributions to the T parameter coming from the Higgs sector. The relevant diagrams are shown in figure 2. The contributions to $T \propto \Pi_{11}(0) - \Pi_{33}(0)$ are similar to the case of the SM, but changing the usual interactions by those of eqs. (2.30)–(2.32). The effective couplings $c_V(p)$, $\tilde{c}_V(p)$ and $c_A(p)$ associated to the external legs are evaluated at zero momentum. Notice that the NGBs ϕ_i are degenerate at tree level. From eq. (2.30) we can see that the one-loop contributions to T from the Feynman diagrams 2-(a) and 2-(b) exactly cancel. We compute now the diagram in figure 1-(c). In order to see explicitly that this contribution is finite, we work in the diagonal basis $\{\gamma, Z\}$. There are two diagrams contributing to Π_{11} , one with a Higgs field h and another with a NGB field ϕ_2 propagating in the loop. This gives:

$$\begin{aligned}
 i\Pi_{11}(0) &= \frac{g^4 v^2}{16} c_A^2(0) \int \frac{d^4 k}{(2\pi)^4} c_A^2(k) G_h G_L \\
 &+ \frac{g^2 g'^2 v^2}{16} \int \frac{d^4 k}{(2\pi)^4} \frac{[g_5^2 c_A(0) (c_V(k) + \tilde{c}_V(k)) + \tilde{g}_5^2 c_A(k)]^2}{(g_5^2 + \tilde{g}_5^2)^2} \\
 &\times G_{\phi_2} (c_w^2 G_\gamma + s_w^2 G_Z),
 \end{aligned} \tag{A.1}$$

where G_{h,ϕ_i} are the propagators of the Higgs and the NGBs and we have factorized the gauge propagators as $G_{\mu\nu}^A = P_{\mu\nu} G_A$. On the other hand, the contribution to Π_{33} is given by:

$$\begin{aligned}
 i\Pi_{33}(0) &= \frac{g^4 v^2}{16} c_A^2(0) \int \frac{d^4 k}{(2\pi)^4} c_A^2(k) G_h G_L \\
 &+ \frac{g^2 g'^2 v^2}{4} c_A^2(0) \int \frac{d^4 k}{(2\pi)^4} c_A^2(k) G_h [s_w^2 G_Z + c_w^2 G_\gamma].
 \end{aligned} \tag{A.2}$$

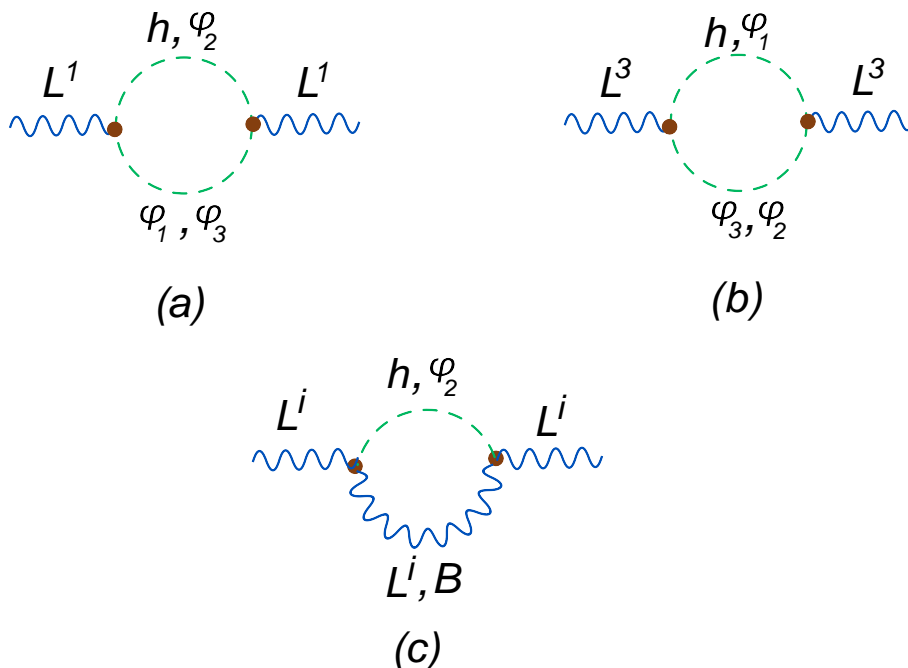


Figure 2: Relevant one-loop diagrams contributing to the T parameter involving the Higgs sector. Here $L^i = L^1, L^3$.

We can see that in the one-loop contributions to the T parameter, proportional to $\Pi_{11}(0) - \Pi_{33}(0)$, the first terms in (A.1) and (A.2) cancel. Also, and just as in the case of the discussion of $S^{1-(c)}$ in section 3.1, the vacuum polarizations are finite. This is because $f_{V,A}(k, z_1)$ are exponentially suppressed at large momentum, $\sim e^{-kz_1}$, implying that also $c_V(k)$, $\tilde{c}_V(k)$ and $c_A(k)$ are. The exponential suppression is due to the Higgs localization in the IR boundary. Had we considered a Higgs with a profile in the bulk, we would have obtained a power suppression. All the gauge propagators can be approximated at large momentum, $kz_1 \gg 1$, by $1/(k^2 \log k)$. Therefore the integrands of eqs. (A.1) and (A.2) are exponentially suppressed for large momentum and the contribution to T from the Feynman diagram 1-(c) is finite.

There are also contributions with only gauge fields running in the loops. Since the difference between the vector and axial correlators is exponentially suppressed at large momentum, these contributions to T are also finite.

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References

- [1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [2] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [[hep-th/9802109](#)];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)].
- [3] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [[hep-ph/0003129](#)].
- [4] Y. Grossman and M. Neubert, *Neutrino masses and mixings in non-factorizable geometry*, *Phys. Lett.* **B 474** (2000) 361 [[hep-ph/9912408](#)].
- [5] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, *Bulk standard model in the Randall-Sundrum background*, *Phys. Rev.* **D 62** (2000) 084025 [[hep-ph/9912498](#)].
- [6] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [[hep-ph/0308036](#)].
- [7] C. Csáki, C. Grojean, L. Pilo and J. Terning, *Towards a realistic model of Higgsless electroweak symmetry breaking*, *Phys. Rev. Lett.* **92** (2004) 101802 [[hep-ph/0308038](#)];
Y. Nomura, *Higgsless theory of electroweak symmetry breaking from warped space*, *JHEP* **11** (2003) 050 [[hep-ph/0309189](#)];
G. Burdman and Y. Nomura, *Holographic theories of electroweak symmetry breaking without a Higgs boson*, *Phys. Rev.* **D 69** (2004) 115013 [[hep-ph/0312247](#)];
G. Cacciapaglia, C. Csáki, C. Grojean and J. Terning, *Oblique corrections from Higgsless models in warped space*, *Phys. Rev.* **D 70** (2004) 075014 [[hep-ph/0401160](#)];
R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. Tanabashi, *The structure of corrections to electroweak interactions in Higgsless models*, *Phys. Rev.* **D 70** (2004) 075008 [[hep-ph/0406077](#)].
- [8] R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson*, *Nucl. Phys.* **B 671** (2003) 148 [[hep-ph/0306259](#)].
- [9] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys.* **B 719** (2005) 165 [[hep-ph/0412089](#)].
- [10] G. Burdman and L. Da Rold, *Electroweak symmetry breaking from a holographic fourth generation*, *JHEP* **12** (2007) 086 [[arXiv:0710.0623](#)].
- [11] A. Falkowski and M. Pérez-Victoria, *Electroweak breaking on a soft wall*, [arXiv:0806.1737](#).
- [12] K. Agashe, C. Csáki, C. Grojean and M. Reece, *The S-parameter in holographic technicolor models*, *JHEP* **12** (2007) 003 [[arXiv:0704.1821](#)].
- [13] PARTICLE DATA GROUP collaboration, C. Amsler et al., *Review of particle physics*, *Phys. Lett.* **B 667** (2008) 1.
- [14] G. Cacciapaglia, C. Csáki, C. Grojean and J. Terning, *Curing the ills of Higgsless models: the S parameter and unitarity*, *Phys. Rev.* **D 71** (2005) 035015 [[hep-ph/0409126](#)].

- [15] G. Degrassi and A. Sirlin, *Gauge dependence of basic electroweak corrections of the standard model*, *Nucl. Phys. B* **383** (1992) 73; *Gauge invariant selfenergies and vertex parts of the Standard Model in the pinch technique framework*, *Phys. Rev. D* **46** (1992) 3104;
G. Degrassi, B.A. Kniehl and A. Sirlin, *Gauge invariant formulation of the S, T and U parameters*, *Phys. Rev. D* **48** (1993) 3963.
- [16] J. Papavassiliou, *Gauge independent transverse and longitudinal self energies and vertices via the pinch technique*, *Phys. Rev. D* **50** (1994) 5958 [[hep-ph/9406258](#)].
- [17] R. Barbieri, B. Bellazzini, V.S. Rychkov and A. Varagnolo, *The Higgs boson from an extended symmetry*, *Phys. Rev. D* **76** (2007) 115008 [[arXiv:0706.0432](#)].
- [18] S. Matsuzaki, R.S. Chivukula, E.H. Simmons and M. Tanabashi, *One-loop corrections to S-parameter in a three site Higgsless model*, *Phys. Rev. D* **75** (2007) 073002 [[hep-ph/0607191](#)];
R.S. Chivukula, E.H. Simmons, S. Matsuzaki and M. Tanabashi, *The three site model at one-loop*, *Phys. Rev. D* **75** (2007) 075012 [[hep-ph/0702218](#)].
- [19] S. Dawson and C.B. Jackson, *Chiral-logarithmic corrections to the S and T parameters in Higgsless models*, *Phys. Rev. D* **76** (2007) 015014 [[hep-ph/0703299](#)].